5. Expected Utility

Duarte Gonçalves

University College London

MRes Microconomics

Overview

1. Decisions under Risk
2. Setup
3. Expected Utility
4. More

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Until now: ignored whether or no DM knows exactly the consequences associated to their actions/choices

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    Computer may or may not be faulty

    Quality control tries to ensure things are fine, but faulty devices exist

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Main questions for today:

(i) obtaining a tractable utility representation of preferences over lotteries (expected utility)

(ii) understanding what EU entails behaviourally and when it is more likely to be a better/worse description of behaviour

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- **Preference relation:** $\succsim \subseteq \Delta(X)^2$.

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- **Probability mixture:** for $\alpha \in [0,1]$ and $p, p' \in \Delta(X)$,
  $\alpha p + (1-\alpha)p' \in \Delta(X)$ denotes lottery s.t.
  $$(\alpha p + (1-\alpha)p')(x) = \alpha p(x) + (1-\alpha)p'(x) \quad \forall x \in X$$
  Note:
  (1) $\Delta(X)$ convex wrt mixtures
  (2) Prob. mixture **is not** a compound lottery/prob. distr. over $\Delta(X)$

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Consider p'' : gain £$10,000$ wp 1/2 and lose £10 wp 1/2.

p'' has far better upside than p and less bad downside than p'; reasonable to expect people to choose p'' over p or p'

Looking for representation that relaxes expected value assumption but retains tractability: separate probability and outcomes

Expected Utility

Definition

$\succsim$ on $\Delta(X)$ has an **expected utility (EU) representation** iff $\exists u : X \to \mathbb{R}$ such that $\forall p, p' \in \Delta(X)$, $p \succsim p' \iff \mathbb{E}_p[u] \geq \mathbb{E}_{p'}[u]$.

u: *Bernoulli* or *von Neumann–Morgenstern utility*

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Continuity of $\succsim$ not sufficient for it to admit EU representation

Independence

Definition

Preference relation $\succsim$ on $\Delta(X)$ sat. **independence** if $\forall p, p' \in \Delta(X)$, $p \succsim (\succ)\, p'$ if and only if $\forall p'' \in \Delta(X)$, and $\forall \alpha \in (0, 1]$, $\alpha p + (1 - \alpha)p'' \succsim (\succ)\, \alpha p' + (1 - \alpha)p''$.

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Independence **necessary** for EU representation $\because$ *expectations are linear in probabilities*

$\mathbb{E}_p[u] = (>)\, \mathbb{E}_{p'}[u] \implies \mathbb{E}_{\alpha p+(1-\alpha)p'}[u] = (>)\, \mathbb{E}_{p'}[u]$ (for $\alpha \in (0,1]$)

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Immediately implies ruling out strict preference for randomisation, i.e., cannot have

$p \sim p'$ and $\alpha p + (1-\alpha)p' \succ p'$.

Continuity

Definition

Preference relation $\succsim$ on $\Delta(X)$ sat.

(i) **Archimedean property** if $\forall p, p', p'' \in \Delta(X)$ s.t. $p \succ p' \succ p''$, $\exists \alpha, \beta \in (0,1)$: $\alpha p + (1-\alpha)p'' \succ p' \succ \beta p + (1-\beta)p''$;

(ii) **vNM continuity** if $\forall p, p', p'' \in \Delta(X)$ s.t. $p \succsim p' \succsim p''$, $\exists \gamma \in [0,1] : \gamma p + (1-\gamma)p'' \sim p'$.

vNM continuity also **necessary** for EU representation:

if $\mathbb{E}_p[u] \geq \mathbb{E}_{p'}[u] \geq \mathbb{E}_{p''}[u]$, then $\exists \gamma \in [0,1]$ s.t. $\gamma \mathbb{E}_p[u] + (1-\gamma)\mathbb{E}_{p''}[u] = \mathbb{E}_{p'}[u]$;

and linearity $\mathbb{E}_p$ wrt p implies

$$\gamma \mathbb{E}_p[u] + (1-\gamma)\mathbb{E}_{p''}[u] = \mathbb{E}_{\gamma p + (1-\gamma)p''}[u].$$

EU Representation

Theorem (von Neumann & Morgenstern 1953)

Let X be finite and $\succsim$ a preference relation on $\Delta(X)$.

(i) $\succsim$ satisfies independence and vNM continuity if and only if it admits an expected utility representation u.

(ii) If u and v are two expected utility representations of $\succsim$, then $\exists \alpha > 0, \beta \in \mathbb{R}$ such that $v = \alpha u + \beta$.

EU Representation

Proof

If part of (i) already discussed. Focus on only if.

Step 1. $\exists \delta_{\overline{x}}, \delta_{\underline{x}} \in \Delta(X)$ such that $\forall \delta_x \in \Delta(X)$, $\delta_{\overline{x}} \succsim \delta_x \succsim \delta_{\underline{x}}$.

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Step 2. $\forall p, p' \in \Delta(X)$ s.t. $p \succsim p'$, $\forall \{p_i\}_{i=1,\dots,n} \subseteq \Delta(X)$, and $\{\alpha_i\}_{i=0,\dots,n} \subset [0,1] : \sum_{i=0}^{n} \alpha_i = 1$, we have

$$\alpha_0 p + \sum_{i \in [n]} \alpha_i p_i \succsim \alpha_0 p' + \sum_{i \in [n]} \alpha_i p_i.$$

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(iii) By independence,

$$\begin{aligned} \alpha_0 p + \sum_{i \in [n]} \alpha_i p_i &= \alpha_0 p + (1-\alpha_0) p'' \\ &\succsim \alpha_0 p' + (1-\alpha_0) p'' = \alpha_0 p' + \sum_{i \in [n]} \alpha_i p_i. \end{aligned}$$

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If $\delta_{\overline{x}} \sim \delta_{\underline{x}}$, set $u = c$ constant; done! (why?)

Otherwise, it must be that $\delta_{\overline{x}} \succ \delta_{\underline{x}}$.

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Proof:

(i) By independence,

$$\left(\frac{\alpha-\beta}{1-\beta}\right)\delta_{\overline{x}} + \left[1 - \left(\frac{\alpha-\beta}{1-\beta}\right)\right]\delta_{\underline{x}} \succ \left(\frac{\alpha-\beta}{1-\beta}\right)\delta_{\underline{x}} + \left[1 - \left(\frac{\alpha-\beta}{1-\beta}\right)\right]\delta_{\underline{x}} = \delta_{\underline{x}}.$$

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(ii) Again by independence,

$$\begin{aligned}\alpha\delta_{\overline{x}} + (1-\alpha)\delta_{\underline{x}} &= \beta\delta_{\overline{x}} + (1-\beta)\left[\left(\frac{\alpha-\beta}{1-\beta}\right)\delta_{\overline{x}} + \left[1-\left(\frac{\alpha-\beta}{1-\beta}\right)\right]\delta_{\underline{x}}\right] \\ &\succ \beta\delta_{\overline{x}} + (1-\beta)\left[\left(\frac{\alpha-\beta}{1-\beta}\right)\delta_{\underline{x}} + \left[1-\left(\frac{\alpha-\beta}{1-\beta}\right)\right]\delta_{\underline{x}}\right] = \beta\delta_{\overline{x}} + (1-\beta)\delta_{\underline{x}}\end{aligned}$$

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Proof:

(i) By Step 3, $\delta_{\overline{x}} \succsim p \succsim \delta_{\underline{x}}$.

(ii) vNM continuity ensures existence of a $\gamma \in [0,1]$.

(iii) By Step 4, it must be unique (why?).

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Step 6. Define $u : X \to \mathbb{R}$ s.t. $u(x) = \gamma(\delta_x)$. Then, $\gamma(p) = \sum_{i\in[n]} p(x_i)\gamma(\delta_{x_i})$.

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Proof: WTS

$$p \sim \left(\sum_{i\in[n]} p(x_i)\gamma(\delta_{x_i})\right) \delta_{\overline{x}} + \left(1 - \sum_{i\in[n]} p(x_i)\gamma(\delta_{x_i})\right) \delta_{\underline{x}}.$$

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By repeated application of independence, Step 2, and definition of γ,

$$\begin{aligned} p = \sum_{i=1}^{n} p(x_i)\delta_{x_i} &\sim \sum_{i=1}^{n} p(x_i)\left[\gamma(\delta_{x_i})\delta_{\overline{x}} + (1-\gamma(\delta_{x_i}))\delta_{\underline{x}}\right] \\ &= \sum_{i=1}^{n} p(x_i)\left(\gamma(\delta_{x_i})\right)\delta_{\overline{x}} + \sum_{i=1}^{n} p(x_i)\left((1-\gamma(\delta_{x_i}))\right)\delta_{\underline{x}} \end{aligned}$$

(i) The claim follows from Step 5.

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Proof:

(i) By Step 4 and Step 5, $\gamma(p)\delta_{\overline{x}} + (1-\gamma(p))\delta_{\underline{x}} \sim p \succsim p' \sim \gamma(p')\delta_{\overline{x}} + (1-\gamma(p'))\delta_{\underline{x}}$, iff $\gamma(p) \geq \gamma(p')$.

(ii) By Step 5 and Step 6, it follows $\mathbb{E}_p[\gamma] = \sum_{i \in [n]} p(x_i)\gamma(\delta_{x_i}) = \gamma(p)$.

(iii) By definition, $\mathbb{E}_p[u] = \mathbb{E}_p[\gamma]$.

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WTS: If u and v are two EU representations of $\succsim$, then $\exists \alpha > 0, \beta \in \mathbb{R}$ s.t. $v = \alpha u + \beta$.

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(vii) Hence, $u = \frac{v - v(\underline{x})}{v(\overline{x}) - v(\underline{x})} \implies v = \alpha u + \beta$, with $\alpha = v(\overline{x}) - v(\underline{x})$ and $\beta = v(\underline{x})$. $\square$

EU Representation

Theorem (von Neumann & Morgenstern 1953)

Let X be finite and $\succsim$ a preference relation on $\Delta(X)$.

(i) $\succsim$ satisfies independence and vNM continuity if and only if it admits an expected utility representation u.

(ii) If u and v are two expected utility representations of $\succsim$, then $\exists \alpha > 0, \beta \in \mathbb{R}$ such that $v = \alpha u + \beta$.

EU representations are unique up to positive affine transformations; cardinal interpretation of u.

Overview

1. Decisions under Risk

2. Setup

3. Expected Utility

4. More
   - Compound Lotteries
   - Issues with Expected Utility

Compound Lotteries

What is a compound lottery?

p: +£5 wp 1/2, -£5 wp 1/2

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ℓ: p wp 1/2, p' wp 1/2

$\ell \neq p''$: +£$5,000$ wp 1/4, +£5 wp 1/4, -£5 wp 1/2

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- Treating compound lotteries and mixtures differently: failure to reduce compound Lotteries
- Turns out that attitudes specific to compound lotteries seem to be closer related to attitudes toward uncertainty than to attitudes toward simple lotteries (e.g., Ortoleva & Dean 2019 PNAS)

Allais Paradox (1953 Ecta)

Paris, sometime between 12 and 17 May 1952, over lunch at conference on choice under risk

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Choosing a) and B) [or b) and A)] cannot be rationalised by EU (why?)

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EU is still a useful model for choice under risk

Understanding better when it holds and when it fails is illuminating

More

Rank-Dependent Expected Utility (Quiggin, 1982 JEBO); cumulative prospect theory (Tversky & Kahneman, 1992 JRU)

- Main gist: small probabilities of the worst events loom larger than they are
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Robustness and Misspecification

- Climate change, limited knowledge, limited modelling capacity
- Variational preferences (Cerreia-Vioglio et al.)